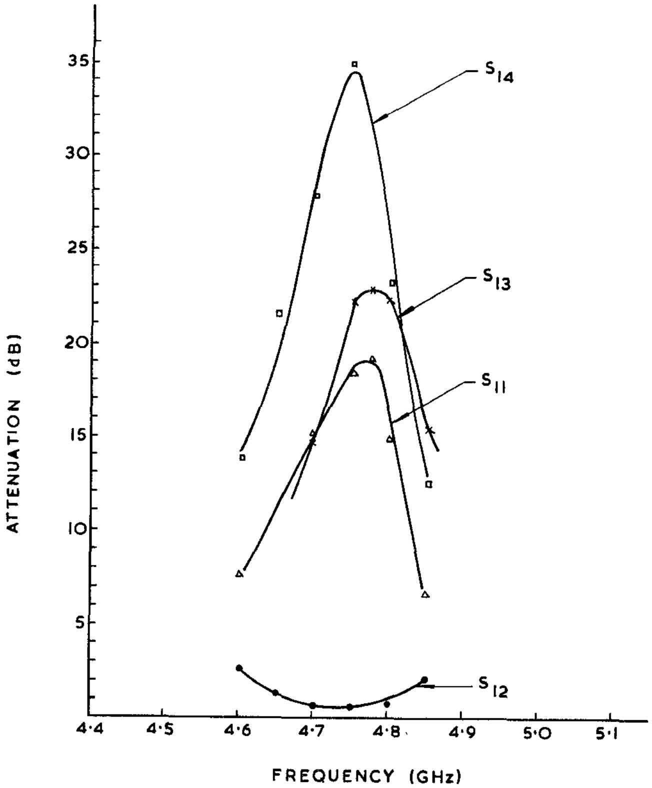

Fig. 8. Scattering parameter of 4-port junction circulator versus frequency.

$$E_z = a_0 + a_3 \cos 3(\tau_3 + \phi)$$

where a_0 and a_3 are arbitrary constants and τ_3 is the phase angle through which the $n = \pm 3$ modes are rotated.

The boundary conditions at the 4 ports are

$$E_z(\phi = 0) = a_0 + a_3 \cos 3\tau_3 = +1$$
$$E_z(\phi = \pi/2) = a_0 + a_3 \cos 3(\tau_3 + \pi/2) = +1$$
$$E_z(\phi = \pi) = a_0 + a_3 \cos 3(\tau_3 + \pi) = 0$$
$$E_z(\phi = 3\pi/2) = a_0 + a_3 \cos 3(\tau_3 + 3\pi/2) = 0.$$

The result is

$$\tau_3 = 15°$$
$$a_0 = 1/2$$
$$a_3 = 1/\sqrt{2}.$$

The ferrite material used in the experimental work was a garnet with a magnetization of 0.05 $\mathrm{Wb/m^2}$ and a dielectric constant of $\epsilon_r = 14.4$. The radius of the ferrite disks was $R = 12.7$ mm and the thickness was $b/2 = 2.54$ mm. The dimensions of the stripline were determined by $W/b = 1.45$ which corresponds approximately to a 50-Ω line for an infinitely thin center conductor. Here, W is the width of the center conductor and b is the ground plane spacing.

Fig. 8 shows the scattering parameters of the junction versus frequency with the device magnetized with a direct magnetic field of 19 kA/m at the frequency at which the $n = \pm 3$ modes are resonant in the demagnetized junction. No external tuning was used to obtain this result. The condition for resonance of the $n = \pm 3$ modes in the demagnetized junction is obtained at the frequency for which $S_{11} = S_{12} = S_{14} = 0$ and $S_{13} = 1$. The frequency at which this last condition is experimentally satisfied coincides with that at which the junction works as a circulator. The experimental radial wavenumber is $kR = 4.75$, which compares with the theoretical value for the $n = \pm 3$ modes of $kR = 4.20$. The experimental radial wavenumber for $n = 0$ in the demagnetized junction was $kR = 4.12$, compared to the theoretical value of 3.83. The $n = \pm 1$, ± 2 modes were also measured for completeness. The results, $kR = 2.06$, compared to 1.84 and $kR = 3.44$ compared with 3.04. The demagnetized permeability was used in obtaining the theoretical wavenumbers [17].

The circulator described in this section should prove attractive using microstrip techniques.

Conclusions

This short paper has described the operation of two 4-port single-junction circulators. Axial junction modes were used in the case of the waveguide device which was established in a systematic way with the help of the eigenvalue approach. A higher order radial mode was used in the stripline case. This allowed one of the adjustment stages to be omitted, leading to a device which consisted of only a simple magnetized disk.

References

[1] L. E. Davis, M. D. Coleman, and J. J. Cotter, "Four-port crossed-waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.* (*1963 Symposium Issue*), vol. MTT-12, pp. 43–47, Jan. 1964.
[2] B. Owen, "The identification of modal resonances in ferrite loaded waveguide Y-junctions and their adjustment for circulation," *Bell Syst. Tech. J.*, vol. 51, no. 3, Mar. 1972.
[3] J. Helszajn and F. C. F. Tan, "Radial line waveguide junction circulators," to be published.
[4] Markuvitz, *Microwave Handbook*. New York: McGraw-Hill.
[5] L. Lewin, *Advanced Theory of Waveguides*. London: Iliffe and Sons, Ltd., 1951.
[6] J. Helszajn, "Three resonant mode adjustment of the waveguide circulator," *Radio Electron. Eng.*, vol. 42, no. 5, May 1972.
[7] J. Helszajn and C. R. Buffler, "Adjustment of the 4-port single-junction circulator," *Radio Electron. Eng.*, vol. 35, no. 6, June 1968.
[8] J. Helszajn, "The adjustment of the *m*-port single-junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 705–711, Oct. 1970.
[9] S. Yoshida, "X circulator," *Proc. IRE* (Corresp.), vol. 47, p. 1150, June 1959.
[10] D. N. Landry, "A single junction four-port coaxial circulator," 1963 WESCON, Part 5, Session 4.2.
[11] C. E. Fay and W. A. Dean, "The four-port single junction circulator in stripline," in *1966 G-MTT Symp. Dig.* (Palo Alto, Calif., May 16–19, 1966), pp. 286–290.
[12] S. R. Longley, "Experimental 4-port E-plane junction circulators," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-15, pp. 378–380, June 1967.
[13] A. G. Bogdanov, "Design of waveguide X-circulators," *Radio Eng. Electron. Phys.* (*USSR*), vol. 14, no. 4, 1969.
[14] J. B. Davies and P. Cohen, "Theoretical design of symmetrical junction stripline circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 506–512, Nov. 1963.
[15] H. Bosma, "On stripline *Y*-circulation at UHF," *IEEE Trans. Microwave Theory Tech.* (*1963 Symposium Issue*), vol. MTT-12, pp. 61–72, Jan. 1964.
[16] J. Watkins, "Circular resonant structures on microstrip," *Electron. Lett.*, vol. 5, pp. 524–525, 1969.
[17] W. E. Courtney, "Analysis and evaluation of a method of measuring the complex permittivity and permeability of microwave insulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 476–485, Aug. 1970.

The Circular Waveguide Step-Discontinuity Mode Transducer

WILLIAM J. ENGLISH

Abstract—**Power conversion coefficients and the launch phase of propagating modes excited by a symmetric step-discontinuity in circular waveguide are accurately predicted by a modal analysis of the discontinuity which includes only a few evanescent modes. The relative power in transmitted and reflected propagating modes is presented as a function of normalized frequency for two step-discontinuity ratios to indicate typical solution results.**

I. Introduction

The use of higher order waveguide modes in a circular radiating aperture for beam shaping and sidelobe control has received attention in recent years [1]–[3]. The inclusion of TM_{11} and TE_{12} modes, along with the fundamental TE_{11} mode, in the radiating aperture permits: a) symmetric radiation patterns; b) low E- and H-plane sidelobes; and c) improved polarization characteristics.

Wexler [4] and Clarricoats [5], [6] have developed a modal analysis approach to waveguide discontinuity problems in which the transverse electromagnetic fields in the discontinuity aperture are expanded in terms of the normal modes of the two connected waveguides, and two simultaneous sets of equations for the complex reflection and transmission coefficients are formed by invoking continuity and zero-field conditions on the transverse components. This short paper summarizes the results of applying this approach to a circular waveguide step-discontinuity mode transducer which is ideally terminated at the source and load.

Manuscript received July 26, 1971; revised May 23, 1973. This short paper is based upon work performed in COMSAT Laboratories under corporate sponsorship.
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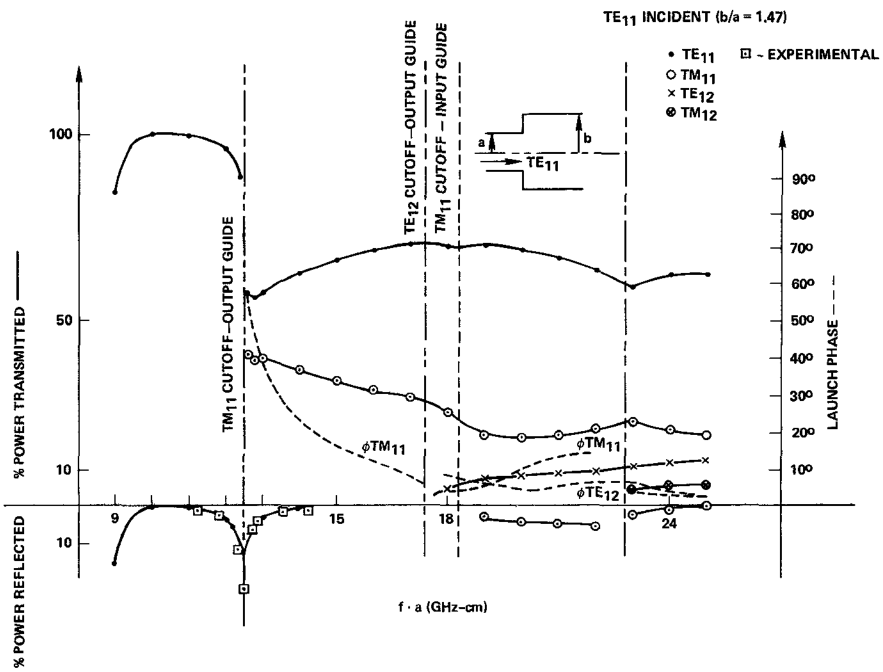

Fig. 1. Mode transducing properties (relative power and launch phase) of a symmetric step-discontinuity waveguide junction illuminated with an incident fundamental TE_{11} mode.

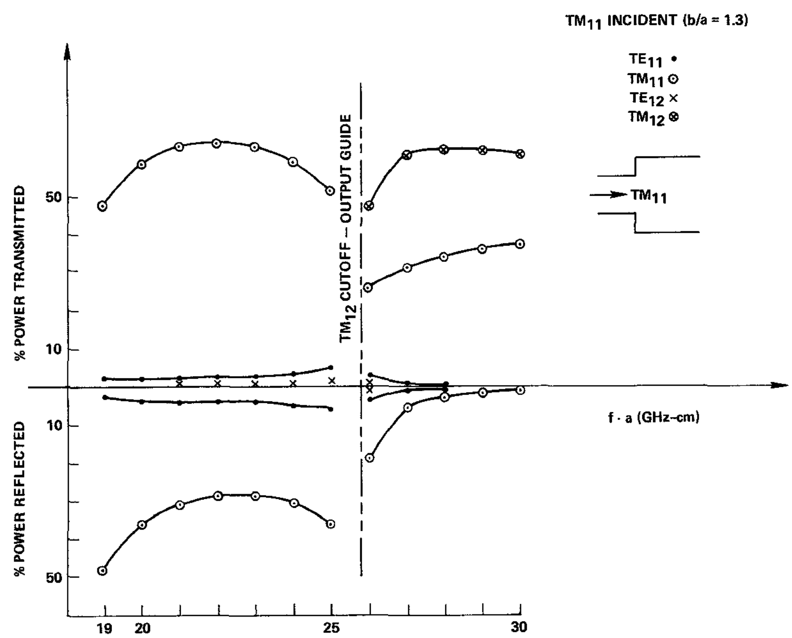

Fig. 2. Mode transducing properties of a symmetric step-discontinuity waveguide junction illuminated with an incident TM_{11} mode.

II. Step-Discontinuity Properties

Fig. 1 illustrates typical mode transducing properties of the step-discontinuity illuminated with a fundamental TE_{11} mode. The relative power in transmitted and reflected propagating modes is presented as a function of normalized frequency for a given step size b/a. The launch phase of the TM_{11} and TE_{12} modes relative to the TE_{11} mode is also shown. The calculated results are in excellent agreement with experimental data previously reported by Agarwal and Nagelberg [7]. In addition, experimentally measured reflection coefficients are shown to confirm that the analysis predicts the high reflection coefficients encountered when the frequency is close to TM_{11} cutoff in the larger output guide. This feature is one of the distinct advantages of the rigorous modal analysis approach which includes reflected wave components in the input waveguide.

Fig. 2 illustrates the mode transducing properties of the junction when it is illuminated with a TM_{11} mode. These properties are informative for the design of multistep transducers. An incident TM_{11} mode only couples strongly to other TM_{1n} modes for the symmetric step-up junction and a large reflected power exists until the TM_{12} mode propagates.

In-phase radial electric field components on each side of the discontinuity aperture are compared in Fig. 3. Ideally, the tangential electric field is continuous across the common aperture area and zero on the metallic portion of the discontinuity aperture. As the number of modes considered in the analysis procedure increases from 10 to 20, the fine structure of the field components continues to change and the ideal boundary and continuity conditions are approached. The radial electric component experiences a singularity at the 90° corner in the smaller guide which is evident in the Fourier series summation.

Convergence results as a function of the number of normal modes

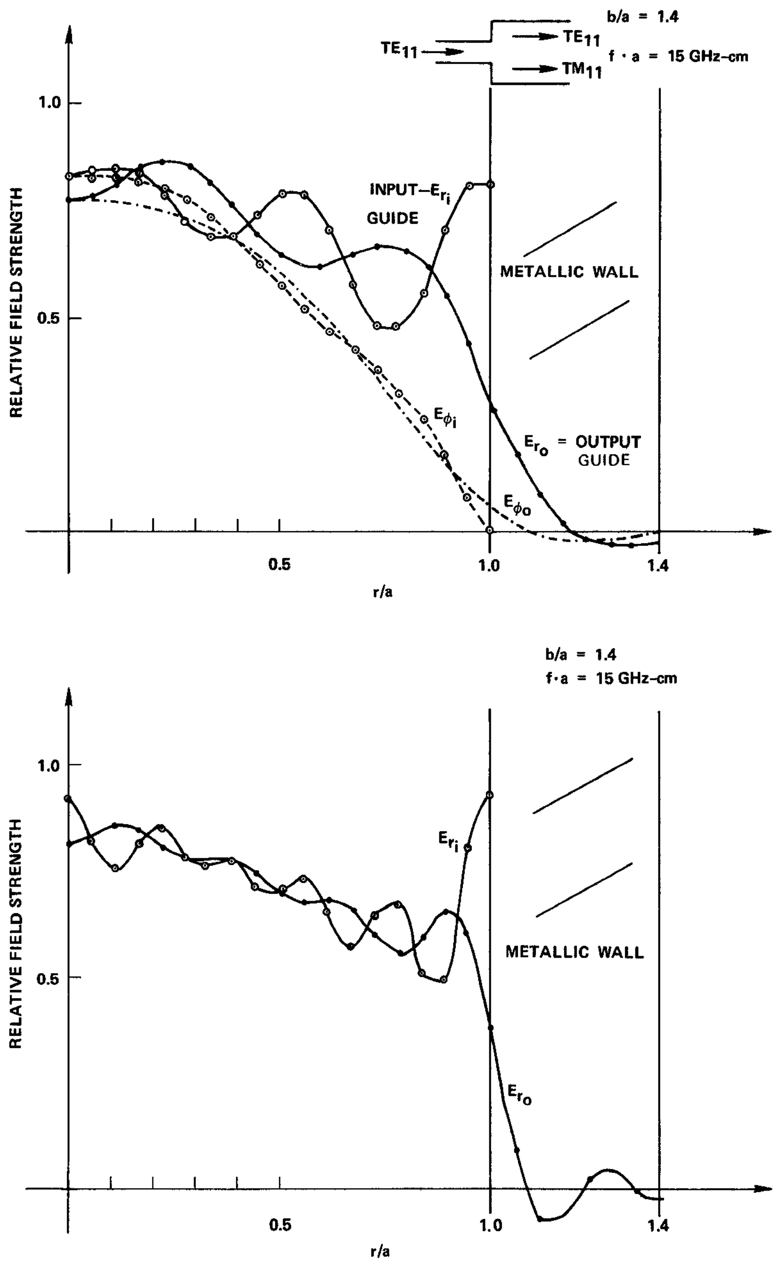

Fig. 3. Comparison of the input and output waveguide transverse aperture fields at the discontinuity junction—in-phase electric components (10 and 20 output guide modes).

TABLE I

JUNCTION PARAMETERS AS A FUNCTION OF THE NUMBER OF NORMAL MODES[a]

Number of Output Guide Modes MT	TE_{11} Transmission Coefficient	Relative TE_{11} Power	TM_{11} Transmission Coefficient	Relative TM_{11} Power
3	0.809 @ -1.88°	0.733	0.386 @ 22.99°	0.265
4	0.790 @ -3.05°	0.699	0.408 @ 20.70°	0.297
5	0.792 @ -2.34°	0.702	0.407 @ 21.42°	0.295
10	0.787 @ -2.48°	0.695	0.411 @ 20.74°	0.302
20	0.785 @ -2.35°	0.691	0.414 @ 20.57°	0.305

[a] *Note:* $b/a = 1.4$, $f \cdot a = 15$ GHz·cm.

TABLE II

RELATIVE POWER IN PROPAGATING AND EVANESCENT MODES[a]

No. of Mode	Reflected Power PROPAGATING	Reflected Power REACTIVE	Transmitted Power PROPAGATING	Transmitted Power REACTIVE
M= 1	PM/P INPUT= 0.003516	+J 0.0	PM/P OUTPUT= 0.691403	+J 0.0
M= 2	PM/P INPUT= 0.0	+J-0.080699	PM/P OUTPUT= 0.305081	+J 0.0
M= 3	PM/P INPUT= 0.0	+J 0.010924	PM/P OUTPUT= 0.0	+J 0.070690
M= 4	PM/P INPUT= 0.0	+J-0.008721	PM/P OUTPUT= 0.0	+J-0.045942
M= 5	PM/P INPUT= 0.0	+J 0.003003	PM/P OUTPUT= 0.0	+J 0.018121
M= 6	PM/P INPUT= 0.0	+J-0.003332	PM/P OUTPUT= 0.0	+J-0.009543
M= 7	PM/P INPUT= 0.0	+J 0.001491	PM/P OUTPUT= 0.0	+J 0.001279
M= 8	PM/P INPUT= 0.0	+J-0.001792	PM/P OUTPUT= 0.0	+J-0.000572
M= 9	PM/P INPUT= 0.0	+J 0.000925	PM/P OUTPUT= 0.0	+J 0.002257
M=10	PM/P INPUT= 0.0	+J-0.001161	PM/P OUTPUT= 0.0	+J-0.003982
M=11	PM/P INPUT= 0.0	+J 0.000667	PM/P OUTPUT= 0.0	+J 0.003604
M=12	PM/P INPUT= 0.0	+J-0.000363	PM/P OUTPUT= 0.0	+J-0.003352
M=13	PM/P INPUT= 0.0	+J 0.000547	PM/P OUTPUT= 0.0	+J 0.001207
M=14	PM/P INPUT= 0.0	+J-0.000738	PM/P OUTPUT= 0.0	+J-0.000402
M=15	PM/P INPUT= 0.0	+J 0.001301	PM/P OUTPUT= 0.0	+J 0.000073
M=16	PM/P INPUT= 0.0	+J-0.001295	PM/P OUTPUT= 0.0	+J-0.000492
M=17	PM/P INPUT= 0.0	+J 0.000658	PM/P OUTPUT= 0.0	+J 0.000926
M=18	PM/P INPUT= 0.0	+J-0.000767	PM/P OUTPUT= 0.0	+J-0.001510
M=19	PM/P INPUT= 0.0	+J 0.000419	PM/P OUTPUT= 0.0	+J 0.000543
M=20	PM/P INPUT= 0.0	+J-0.000517	PM/P OUTPUT= 0.0	+J-0.000815

[a] *Note:* $b/a = 1.4$, $f \cdot a = 15$ GHz·cm.

considered are presented in Table I for a frequency where TE_{11} and TM_{11} modes both propagate in the output guide. Mode power conversion and launch phase coefficients converge rapidly and are essentially constant after six to eight evanescent modes on each side of the discontinuity are included. The ratio of the number of input to output waveguide modes considered is not significant if more than six evanescent modes are included; this supports a similar conclusion by Clarricoats [6].

The relative reactive power in the evanescent modes is shown in Table II, in which 20 modes are considered. The modes are numbered in terms of increasing cutoff frequencies. The reactive power quickly diminishes as the mode order increases and the inductive or capacitive nature of the discontinuity is provided by the total reactive power.

ACKNOWLEDGMENT

The author wishes to thank R. W. Gruner for his helpful discussions and his assistance with the measurement apparatus.

REFERENCES

[1] P. D. Potter, "A new horn antenna with suppressed sidelobes and equal beamwidths," *Microwave J.*, pp. 71–78, June 1963.
[2] J. S. Ajioka and H. Harry, Jr., "Shaped beam antenna for earth coverage from a stabilized satellite," *IEEE Trans. Antennas Propagat.*, vol. AP-18, pp. 323–327, May 1970.
[3] S. Drabowitch, "Multimode antennas," *Microwave J.*, pp. 46–51, Jan. 1966.
[4] A. Wexler, "Solution of waveguide discontinuities by modal analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 508–517, Sept. 1967.
[5] P. J. B. Clarricoats and K. R. Slinn, "Numerical solution of waveguide discontinuity problems," *Proc. Inst. Elec. Eng.*, vol. 114, pp. 878–885, July 1967.
[6] P. H. Masterman and P. J. B. Clarricoats, "Computer field-matching solution of waveguide transverse discontinuities," *Proc. Inst. Elec. Eng.*, vol. 118, pp. 51–63, Jan. 1971.
[7] K. K. Agarwal and E. R. Nagelberg, "Phase characteristics of a circularly symmetric dual-mode transducer," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-18, pp. 69–71, Jan. 1970.

Radiation Loss from Open-Circuited Dielectric Resonators

J. WATKINS

***Abstract*—The Q factor of an open-circuited resonator is influenced by dielectric, conductor, and radiation losses. This short paper discusses these losses and shows that insight into the radiation loss can be obtained by an extension to the analysis given by Lewin. This shows that the radiation loss is a maximum for the dominant mode and that provided the permittivity is not less than 9.0 the radiation losses are at a minimum for the second mode.**

It is suggested that these results may be applied to the design of filters based on dielectric resonators. The application of the results to stripline resonators in which the dielectric extends past the termination of the upper conducting strip is more difficult, and it is suggested that experimental work is required to investigate postulated improvements. Finally, some of the radiation patterns of the open-circuited dielectric resonators, obtained in this paper, show interesting directional properties which may be applied to the design of antenna systems.

Manuscript received November 3, 1972; revised May 21, 1973. This work was carried out under a contract sponsored by the U. K. Science Research Council, in the Department of Electronics, Chelsea College, University of London.
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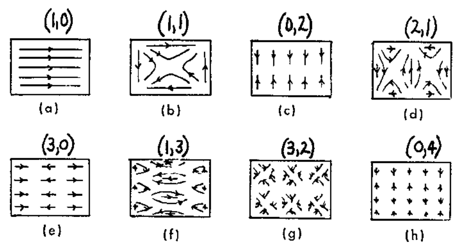

Fig. 1. Patterns of the conduction current for various modes.

I. INTRODUCTION

Resonators involving ceramic substrates whose upper and lower surfaces have been coated with metallic conductors have previously been reported by Napoli and Hughes [1]. To avoid the degenerate resonances which would occur with a square resonator, a rectangular geometry is chosen and simple theory shows that

$$\mathcal{f} = \frac{c}{2L_z} \frac{1}{\sqrt{\epsilon_r}} \left(m^2 + \frac{n^2}{\gamma^2} \right)^{1/2} \tag{1}$$

where

$\mathcal{f}$	resonant frequency;
c	velocity of light;
ϵ_r	relative permittivity;
L_z	larger side;
γL_z	shorter side;
γ	<1;
m, n	integers.

The assumption is made here that there are no variations in the fields across the thickness of the dielectric. Fig. 1(a)–(h) shows some of the patterns of the conduction current for resonances with different combinations of m and n. Fig. 2 shows the dimensions of the resonator and the coordinate system used in this paper.

The Q factor of these resonances are due to losses from four possible mechanisms: 1) coupling with the load and generator; 2)